SELF-REGULATED ACCRETION DISKS

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ABSTRACT

We consider a class of fully self-gravitating accretion disks, for which efficient cooling mechanisms are assumed to maintain the disk close to the margin of Jeans instability. For such self-regulated disks the equations become very simple in the outer regions, where the angular momentum convective transport approximately balances the viscous transport. These latter equations are shown to lead naturally to a self-similar solution with flat rotation curve, with circular velocity proportional to $(\dot{M})^{1/3}$ and essentially fixed opening angle.

Subject headings: accretion, accretion disks — gravitation — hydrodynamics

1. Introduction

Studies of accretion disks have generally been motivated by astrophysical phenomena in environments, such as X-ray binaries, where the presence of a massive central object is established and dominates the scene. Under these circumstances, the gravitational field in the accretion disk is basically Keplerian and the self-gravity of the disk is expected to play a secondary role (e.g., see Pringle 1981). On the other hand, there are regimes where the role of self-gravity is important, so that the standard models require modification. One step in this direction is taken by considering the effects of self-gravity on the vertical structure of the disk (Paczyński 1978) or by focusing on the impact of self-gravity on waves and on transport processes (e.g., Lin & Pringle 1987; Adams et al. 1989). In particular, the role of Jeans instability in the disk has sometimes been taken into account by modifying the viscosity prescription (see Eqs.(14) and (16) of the paper by Lin & Pringle 1990). In any case, much like in certain studies of planetary rings, it is still customary to retain the assumption that the gravitational field that sets the angular velocity of the disk is basically Keplerian and spherically symmetric, even though there are indications that under certain circumstances (e.g., for protostellar disks; see Lin & Pringle 1990; Drimmel 1996; and references therein) the disk mass may be significant.

In contrast with the case of planetary rings, in galaxy disks self-gravity plays an additional non-trivial role, which is the shaping of the underlying rotation curve (e.g., see van Albada & Sancisi 1986). By analogy, one may imagine conditions, for protostars or protogalaxies, where the disk forms *without* a prominent central object, and accretion initially proceeds mostly under the action of gravity within the disk alone. These are precisely the conditions that this paper is devoted to.

The construction of models of accretion disks is generally based on the specification of the relevant radiation processes associated with the disk material, which gives a direct

connection with the available observational data. Unfortunately, since the data give only indirect clues on the microscopic state of the disk, one has to resort to highly idealized models with a number of free parameters. For the energy transport, viscous heating is determined from the equations of hydrodynamics and radiation cooling is calculated under a variety of assumptions on the disk opacity and on the radiation mechanisms involved. The models are often based on a polytropic equation of state. In turn, the amount of viscosity that is demanded by the observed phenomena explained in terms of accretion disks is too large, well beyond that estimated from the classical expressions for the viscosity coefficients derived from gas dynamics or plasma physics. Then one is led to imagine that in the real accretion disks collective phenomena are at work, producing "anomalous transport". This is not too surprising, since the inadequacy of the classical transport theory is well known in the context of laboratory and space plasmas (e.g., see Coppi 1980). The traditional way to bypass such a stumbling block is to give a physically intuitive prescription for the viscosity, where our ignorance of the underlying physical mechanisms is hidden in a dimensionless parameter called α (Shakura & Sunyaev 1973). This also opens the way to a positive confrontation with the observations. It is then hoped that an examination of the available physical mechanisms (e.g., see Balbus & Hawley 1991) may eventually identify the origin of the large viscosity required.

While the momentum transport equations are drastically simplified by means of the α-prescription, in general the energy equations are kept in their ideal form and followed in detail down to their full consequences (e.g., see the equations describing advection-dominated accretion; Popham & Narayan 1991; Narayan & Yi 1994). One may really wonder whether a situation such as that of a protogalaxy or of a protostar, where the cold infalling material may include gas clouds of different sizes and different internal temperatures (in atomic or molecular form), dust, and particulate objects (as in planetary rings), should be described by a one-component polytropic equation of state or whether,

instead, such a complex collective system should be modeled by a completely different approach, more in line with the spirit that has led to the α-prescription for the viscosity.

The purpose of this paper is to explore the consequences of one such an alternate approach. We study a class of accretion disks where self-gravity plays a full role and the energy equation is replaced by a self-regulation prescription associated with the onset of Jeans instability in the disk. A complete set of equations is thus proposed for the description of a class of steady-state models. Surprisingly, the set of equations immediately leads to a simple self-similar solution, with the disk density inversely proportional to the radial distance and with flat rotation curve. The opening angle set by the ratio h/r is essentially fixed, while all the other properties of the disk depend only on the value of α and on the value of the accretion rate $\dot{M}$.

2. Basic Equations for Self-Regulated Disks

We consider a steady-state axisymmetric viscous inflow of a thin disk, with constant mass ($\dot{M}$) and angular momentum ($\dot{J}$) accretion rates

$$\dot{M} = -2\pi r \sigma u \quad , \tag{1}$$

$$\dot{J} = \dot{M} r^2 \Omega + 2\pi \nu \sigma r^3 \frac{d\Omega}{dr} \quad . \tag{2}$$

Here r is the radial coordinate in the plane of the disk, σ is the disk density, u is the radial velocity (so that an inflow, $u < 0$, corresponds to a positive value of $\dot{M}$), Ω is the local angular velocity of the disk, and ν is the relevant viscosity coefficient. Note that for the common situation of a negative shear, $d\Omega/dr < 0$, the viscous contribution to the angular momentum transport is negative, i.e. in the outward direction.

For a cool, slowly accreting disk the radial momentum balance equation requires

$$\Omega^2 \sim \frac{1}{r}\frac{d\Phi_\sigma}{dr} + \frac{GM_\star}{r^3} \quad , \tag{3}$$

where $M_\star$ is the mass of the central object and Φ_σ is the contribution to the gravitational field provided by the mass of the disk

$$\Phi_\sigma(r) = -2\pi G \int_0^\infty K^{(0)}(r,r')\sigma(r')dr' \quad , \tag{4}$$

with $K^{(0)}(r,r') = (1/\pi)\sqrt{r'\zeta/r}K(\zeta)$. Here $\zeta \equiv 4rr'/(r+r')^2$ and $K(\zeta)$ is a complete elliptic integral of the first kind. [Corrections to Eq.(3) related to the disk pressure gradient and to the convective gradient $u(du/dr)$ can be easily incorporated, but do not change the following discussion.]

The viscosity in the disk is taken according to the Shakura & Sunyaev (1973) prescription

$$\nu = \alpha c h \quad , \tag{5}$$

where the half-thickness h is related to the effective thermal speed c following the hydrostatic requirement of a self-gravitating disk

$$h = \frac{c^2}{\pi G \sigma} \quad . \tag{6}$$

Note that inserting (6) into (5) yields the simple relation $\nu\sigma = (\alpha/\pi G)c^3$, which can be inserted further in Eq.(2) to give

$$G\dot{J} = G\dot{M}r^2\Omega + 2\alpha c^3 r^3 \frac{d\Omega}{dr} \quad . \tag{7}$$

Even with α treated as an "assigned parameter", the equations written so far form an incomplete set. As mentioned in the Introduction, the set is usually closed by means of an energy equation, or by an equation balancing viscous heating and radiation losses. Here we explore the possibility of closing the set by a self-regulation prescription (see Bertin 1991) related to the Jeans instability of the disk. The physical argument is that, in the presence of Jeans instability, on the fast dynamical timescale the disk is bound to heat up until it becomes marginally stable; if the disk is warmer to begin with, it is assumed that dissipation, e.g. in the form of inelastic collisions of the fluid elements of the disk, make the effective thermal speed c quickly drop to the marginal value. The condition of marginal stability is of the form

$$\frac{c\kappa}{\pi G\sigma} = \bar{Q} \quad , \tag{8}$$

with $\bar{Q} \approx 1$. Here κ is the epicyclic frequency.

Massive self-gravitating disks with $c\kappa/\pi G\sigma$ close to unity are known to be subject to violent spiral instabilities (e.g., see Bonnell 1994, Laughlin & Bodenheimer 1994; for the case of galaxy disks, see Bertin et al. 1989). Therefore, disks of the kind considered in this paper may well be in a situation where the gravitational torques associated with non-axisymmetric instabilities are able to contribute a significant amount of "anomalous viscosity". In this case, there might be no need for other mechanisms (such as those, often invoked, related to magnetic fields) to drive the overall accretion process in the disk. The hope is that the axisymmetric model described here, while obviously oversimplifying the actual state of the disk, is able to capture the basic dynamical properties of a fully self-gravitating accretion process.

In view of the above remarks, it would be desirable to add one further relation, between α and $\bar{Q}$, in order to properly incorporate the gravitational instability contributions to the viscosity prescribed by Eq.(5). Previous studies (Lin & Pringle 1987, 1990; Laughlin & Bodenheimer 1994) have focused on the role of self-gravity on viscous transport and indicate that there is some merit in an axisymmetric model with modified viscosity. Here we take a drastically different approach in that we retain the standard functional dependence of the viscosity (see Eq.(5)) and we use the marginal condition (8) to replace the energy equations altogether. As to the use of the α-prescription, it has been noted (see Laughlin & Różyczka 1996, who based their analysis on hydrodynamical simulations of a polytropic thin disk) that α should be allowed to vary with radius and that the prescription may be inadequate *a priori*, since it is inherently local, while the self-excited non-axisymmetric modes involved are global. In the absence of a convincing global model, we have decided to explore first the consequences of the simple constant-α case. *A posteriori*, at least for the *self-similar solution* described in the next section, this simplifying assumption is quite natural and may well turn out to be dynamically consistent.

For specified values of α and $\bar{Q}$, equations (3), (7), and (8) thus form a complete set (we take Φ_σ defined by (4)) with solutions determined by three physical parameters, i.e. $M_\star$, $\dot{J}$, and $\dot{M}$. One may argue that at large radii the effects of $M_\star$ and $\dot{J}$ eventually become unimportant and thus deal with simple (approximate) relations that are appropriate for the outer disk. Thus we are led to the following reference case.

3. Flat Rotation Curve

In this paper we focus on the special case where $M_\star = 0$ and $\dot{J} = 0$. The basic equations are then reduced to

$$\Omega^2 = \frac{1}{r}\frac{d\Phi_\sigma}{dr} \quad , \tag{9}$$

$$2\alpha c^3 = G\dot{M}\left|\frac{d\ln\Omega}{d\ln r}\right|^{-1} \quad , \tag{10}$$

$$\left(\frac{c}{\pi G\sigma}\right) 2\Omega \left(1 + \frac{1}{2}\frac{d\ln\Omega}{d\ln r}\right)^{1/2} = \bar{Q} \quad , \tag{11}$$

where, we recall, we have in mind $\dot{M} > 0$, $0 < \alpha \lesssim 1$, and $\bar{Q} \approx 1$. The equivalent thermal speed c can be eliminated to give a relation between density and angular velocity

$$\pi G\sigma = \frac{2\Omega}{\bar{Q}}\left(1 + \frac{1}{2}\frac{d\ln\Omega}{d\ln r}\right)^{1/2}\left(\frac{G\dot{M}}{2\alpha}\right)^{1/3}\left|\frac{d\ln\Omega}{d\ln r}\right|^{-1/3} \quad . \tag{12}$$

Therefore an obvious natural solution is provided by the self-similar mass distribution characterized by

$$2\pi G\sigma r = r^2\Omega^2 = V^2 = const \quad , \tag{13}$$

which solves both (12) and the self-consistency condition (9). Thus we find

$$c = \left(\frac{G\dot{M}}{2\alpha}\right)^{1/3} \approx \left(\frac{27\dot{M}}{4\pi\alpha}\right)^{1/3} \quad 10 \ km/sec \quad , \tag{14}$$

$$V = \frac{2\sqrt{2}}{\bar{Q}}c \quad , \tag{15}$$

$$u = -\frac{\alpha\bar{Q}^2}{4}c \quad , \tag{16}$$

$$\frac{h}{r} = \frac{\bar{Q}^2}{4} \quad , \tag{17}$$

$$\nu\sigma = \frac{\dot{M}}{2\pi} \quad . \tag{18}$$

The self-consistent solution provided above is a generalization of the self-similar solution for self-gravitating disks (Mestel 1963) to the case of viscous disks. It has infinite mass and a singularity at $r = 0$, which is even more evident here in the viscous case, since a constant accretion rate implies a constant accumulation of mass at the origin. Since the value of $\bar{Q}$ is well constrained by dynamics, the final solution depends only on the value of the parameter α and on the accretion rate $\dot{M}$. Note that in the last expression of Eq.(14) $\dot{M}$ is to be given in units of $M_{\odot}/yr$. One interesting feature is that the opening angle of the disk is fixed ($\approx$ 14 degrees for $\bar{Q} = \sqrt{2}/2$), set by Eq.(17), independent of α and of the mass accretion rate.

4. Discussion

Here below we list a few points that we plan to investigate in a following article. A resolution of these issues should be able to bring down the singular self-similar disk to more realistic conditions.

(i) Within the assumptions of the previous Section, we note that values of $\bar{Q}$ significantly larger than unity would be undesired, because they go against the thin disk approximation (cf. Eqs.(15) and (17)). However, thickness itself helps to cure the problem; it is possible to show that the effects of finite thickness reduce $\bar{Q}$ from its reference value of unity down to $\approx \sqrt{2}/2$. On the other hand, a more thorough investigation of the plausible range of $\bar{Q}$ in a self-regulated disk is desired.

(ii) The singularity at $r = 0$ should be removed, especially since, for a large class of systems, the central region is the astrophysically interesting part of the disk. Relaxing the assumptions of $\dot{J} = 0$ and $M_\star = 0$ will certainly open the way to models that have a better physical justification. In particular, a smooth, self-regulated process is better justified outside the extreme regime where all the mass is in the disk. Note that, for given values of $M_\star$, $\dot{M}$, and α, the rotation curve will be modified at radii smaller than or of the order of $r_S \equiv (GM_\star/4)(G\dot{M}/2\alpha)^{-2/3}$. Furthermore, independently of the mass of the central object, we expect a change in the properties of the solution on a scale defined by $r_J \equiv (|\dot{J}|/4\dot{M})(G\dot{M}/2\alpha)^{-1/3}$. A number of regimes are available depending on the relative size of the two lengthscales r_S and r_J. We recall that the equations studied in the previous Section are approximately correct at large radii, i.e. at $r > r_S$, $r > r_J$.

(iii) For specific astrophysical systems, one should try to set also a proper boundary condition at large radii, and check to what extent an approximately self-similar solution can be brought to match the physical conditions there. For example, in the context of binary stars where one star supplies mass to the other, while there may be a significant radial range available for the equations of the previous Sections to be approximately correct, r should not exceed the relevant Roche radius.

(iv) Especially from the expression for the thermal speed (14), it is clear that, unless unrealistic values for the accretion rate are considered, the self-similar solution calculated above has very little to say about X-ray emitting astrophysical disks. In view of this difficulty, it is important to explore the possibility that in some radial range, presumably close to the center, the self-regulation constraint may break down or, rather, be replaced or overruled by a different energy balance equation. In turn, this would open the possibility of hotter disks and, in any case, of disks for which the "rigid" structure of the self-similar solution is relieved. [In spiral galaxies, the self-regulation process is expected to take place

in the outer disk, but generally to break down at small radii, where c increases significantly above the marginal value in most realistic models (see Bertin et al. 1989).]

While we were working on the natural follow-up extensions (i) - (iv) of the self-similar solution found in Section 3, we have noted that self-similar, self-gravitating disks have been discussed, very recently, by Mineshige & Umemura (1996). In spite of the apparent similarity of their solution to ours (they also give a prominent role to self-gravity both in the vertical and in the horizontal directions and they also find Mestel solutions for the mass distribution), there are significant differences between their work and the present article, especially at the physical level. In particular, in their article: (a) The starting point is an entropy transport equation (instead of our condition (8)); (b) The self-similar solution is sought from the beginning, then their finding that for a self-gravitating disks the only power-law solutions are those with $r\Omega = const$ is not surprising (while in our Section 3 the flat rotation curve turns out, *a posteriori*, to be the natural solution for a steady-state self-regulated disk); (c) Isothermality is forced by requiring the disk density to follow the behavior of the Mestel disk, but is inconsistent with possible cooling scenarios (either optically thin free-free emission or black-body radiation) that determine the entropy transport equation (our solution is instead internally consistent); (d) The opening angle of the disk (i.e., h/r) is a free parameter (while it is fixed in the present paper; see Eq.(17)); (e) The realization that there is a simple, dimensionally interesting connection between the thermal speed c in the self-similar disk and the accretion rate (see our Eq.(14)) is apparently missed.

In conclusion, based on the assumption that the self-gravity of the disk plays a dominant role, we have identified here a simple analytical solution for self-regulated, steady-state accretion disks that appears to be an interesting starting point for the construction of a new class of accretion disks. Quite unexpectedly, we have shown that by

promoting self-gravity to its full role one is naturally led to flat rotation curves.

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